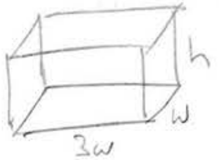
[2]



MAXIMIZE V = VOLUME OF BOX

BY CHANGING w = WIDTH OF BOX

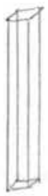
$V = 3w^2h$

$\text{COST} = 4(3w^2) + 5(3wh + wh + 3wh + wh) + 6(3w^2)$

$= 30w^2 + 40wh = 270$

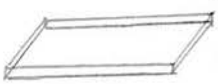
$3w^2 + 4wh = 27$

$h = \dfrac{27 - 3w^2}{4w}$



$V = \dfrac{3w^{\cancel{2}}(27 - 3w^2)}{4\cancel{w}} = \dfrac{9}{4}(9w - w^3)$

MIN w : $w > 0$



MAX w : $30w^2 < 270 \rightarrow -3 < w < 3$

$w \in (0,3)$

IF $w=0$ OR $w=3$, $V=0$ IE. NOT MAX

SO USE $w \in [0,3]$

$V' = \frac{9}{4}(9 - 3w^2)$ EXISTS ON $[0,3]$

$= 0$ IF $w = \sqrt{3} \in [0,3]$

$V(0) = 0$

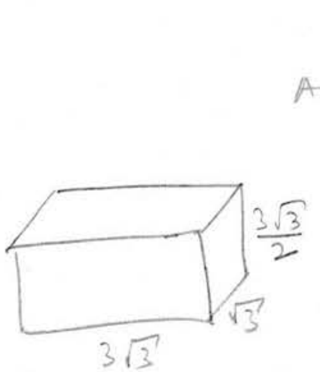
$V(\sqrt{3}) = \frac{9}{4}(9\sqrt{3} - 3\sqrt{3}) = \frac{9}{\cancel{4}_2}(\cancel{6}^3\sqrt{3}) = \dfrac{27\sqrt{3}}{2}$

$V(3) = 0$

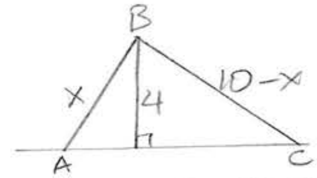
THE LARGEST BOX HAS WIDTH $\sqrt{3}$ FT

LENGTH $3\sqrt{3}$ FT

AND HEIGHT $\dfrac{27-9}{4\sqrt{3}} = \dfrac{\cancel{18}^{9}}{\cancel{4}_{2}\sqrt{3}} = \dfrac{3\sqrt{3}}{2}$ FT



[3] MINIMIZE y = LENGTH OF AC

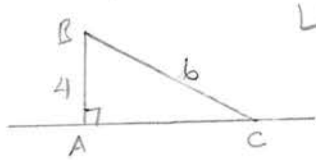


BY CHANGING x = LENGTH OF AB

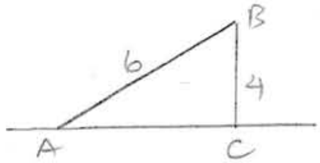
$y = \sqrt{x^2-16} + \sqrt{(10-x)^2-16}$ = LENGTH OF A TO PERPENDICULAR + LENGTH OF PERPENDICULAR TO C

NOTE: IF $x^2-16<0$ OR $(10-x)^2-16<0$, y DNE SO $x^2-16 \geq 0$ AND $(10-x)^2-16 \geq 0$ ON DOMAIN

MIN x : $x=4$



MAX x : $x=6$



$x \in [4,6]$

$$y' = \frac{2x}{2\sqrt{x^2-16}} + \frac{2(10-x)(-1)}{2\sqrt{(10-x)^2-16}}$$

DNE
IF $x^2-16=0$ IE. $x = \cancel{-4}, 4 \in [4,6]$
OR
IF $(10-x)^2-16=0$
$10-x = \pm 4$
$x = 10 \pm 4 = \cancel{14}, 6 \in [4,6]$

$$\frac{x}{\sqrt{x^2-16}} - \frac{10-x}{\sqrt{(10-x)^2-16}} = 0$$

$$\frac{x}{\sqrt{x^2-16}} = \frac{10-x}{\sqrt{(10-x)^2-16}}$$

$$\frac{x^2}{x^2-16} = \frac{(10-x)^2}{(10-x)^2-16}$$

$$\cancel{x^2(10-x)^2} - 16x^2 = \cancel{x^2(10-x)^2} - 16(10-x)^2$$

$\pm x = 10-x$

$\cancel{-x} = 10 \cancel{-x}$ $\quad$ $x = 10-x \rightarrow x=5$

IMPOSSIBLE

$y(4) = \sqrt{20}$
$y(5) = 3+3 = 6 = \sqrt{36}$
$y(6) = \sqrt{20}$

THE SHORTEST DISTANCE IS $2\sqrt{5}$